I. introduce PD stuff.

II. Define crystalline coh. "by hand".

III. Define crystalline site.

Defn. A PD ring is a triple (A, I, γ) where $I \subseteq A$ is an ideal.

and γ is a system of set maps $\gamma_i : I \longrightarrow A$ for $i \geq 1$, s.t.

0) (convention) $\gamma_0(x) = 1$, $\gamma_{-i}(x) = 0$ $\forall i \geq 1$.

1) $\gamma_1(x) = x$. $\gamma_i(x) \in I$ if $i \geq 1$.

2) $\gamma_k(x+y) = \sum_{i+j=k} \gamma_i(x)\gamma_j(y)$, $\forall x, y \in I$

3) $\forall \lambda \in A$, $x \in I$, $\gamma_k(\lambda x) = \lambda^k \gamma_k(x)$.

4). $x \in I$, $\gamma_i(x)\gamma_j(x) = ((i,j))\gamma_{i+j}(x)$ where $((i,j)) = \frac{(i+j)!}{i!\,j!}$.

5) $\gamma_p(\gamma_q(x)) = \frac{(pq)!}{p!(q!)^p}\gamma_{pq}(x)$.

Rmk.

- γ is said a PD structure on I.
- Morally "$\gamma_i(x) = \frac{x^i}{i!}$". If γ is obvious from context, we denote $\gamma_i(x) = x^{[i]}$
- 0)+1) + 4) $\Rightarrow$ $n!\gamma_n(x) = x^n$ $\forall n \geq 0$. & 3) $\Rightarrow$ $\gamma_k(0) = 0$ $\forall k \geq 1$.

morphism $(A, I, \gamma) \longrightarrow (B, J, \delta)$ is what you think it's.

e.g. $\mathcal{O}$ - DVR, w/ $p = u \cdot \pi^e$ w/ $e \leq p-1$ for all prime p.

Lemmata (A, I, γ)

- if $m \cdot A = 0$ for some $m \in \mathbb{N}$ $\Longrightarrow$ I is locally nilpotent ($\forall x \in I$, $x^{p^m} = 0$ ~~for some m~~)
- if $J \subseteq I$ is a PD subideal. (i.e. $\gamma_r(j) \in J$ $\forall j \in J$, $r \geq 1$).

then $(A/J, I/J, \bar{\gamma})$ is also a PD structure.

& $A \longrightarrow A/J$ is a PD morphism.

- direct limit of PD w/ $\underbrace{\text{PD morphisms}}_{\text{transition}}$ is still PD.

• If $\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \\ C & & \end{array}$, $I \subseteq B$, $J \subseteq C$ s.t. $B \twoheadrightarrow B/I$ ($\exists$ section), $C \twoheadrightarrow C/J$ ($\exists$ section)

then $K = \ker\left(B \otimes_A C \twoheadrightarrow B/I \otimes_A C/J\right)$ has a unique PD structure compatible w/ (B, I) & (C, J).

• I is a PD ideal $\Rightarrow$ $I^n \subseteq I$ PD subideal

e.g. $(W(k), (p)) \Rightarrow (W_n(k), (\overline{p}))$ has a natural PD structure.

• Let A be a ring, M be an A-module.

$\exists$ $\Gamma_A(M) = A \oplus M \oplus \Gamma_2(M) \oplus \cdots$ w/ $\Gamma_A^+(M) = M \oplus \Gamma_2(M) \oplus \cdots$

having PD structure γ s.t.

$$\mathrm{Hom}_A(M, J) = \mathrm{Hom}_{PD}\left((\Gamma_A(M), \Gamma_A^+(M), \gamma), (B, J, \delta)\right)$$

$\forall$ PD ~~ring~~ A-alg. (B, J, δ).

~~well behaved~~ preserved under base change, colimit.

turn finite ~~direct sum~~ $\oplus$ into ~~tensor~~ $\otimes$.

if $M = \bigoplus_I A \cdot x_i$. $\Gamma_A(M) =: A\langle x_i \,;\, i \in I\rangle$ is called the divided polynomail alg. in $\{x_i, i \in I\}$.

Defn. ① given (A, I, γ), $A \longrightarrow B$,

we say "γ extends to B" if $\exists$ $\overline{\gamma}$ on $I \cdot B$ s.t.

$(A, I, \gamma) \longrightarrow (B, IB, \overline{\gamma})$ is a PD morphism.

② (A, I, γ). $A \to B$ is said cptible w/ PD structure if Given (B, J, δ). γ extends to $\overline{\gamma}$ & $\overline{\gamma} = \delta$ on $IB \cap J$.

equivalently $\exists$ PD structure α on $IB + J$ s.t.

$$(A, I, \gamma) \xrightarrow{PD} (B, IB + J, \alpha) \xleftarrow{PD} (B, J, \delta).$$

Thm. (PD envelope). Given (A, I, γ), $A \to B$, $J \subseteq B$ ideal.

Then $\exists!$ a B-alg. $D_{B,\gamma}(J)$ w/ PD ideal $(\bar{J}, [n])$.

s.t. ① $J \cdot D_{B,\gamma}(J) \subseteq \bar{J}$

② $[\cdot]$ is cptible w/

$$(A, I, \gamma) \longrightarrow (D_{B,\gamma}(J), \bar{J}, [\cdot]).$$

③ universal property: $\forall$ $(A, I, \gamma) \xrightarrow{\text{cptible}} (C, K, \delta)$.

$$\mathrm{Hom}_{PD}\big((D_{B,\gamma}(J), \bar{J}, [\cdot]), (C, K, \delta)\big) = \mathrm{Hom}_{A}\big((B,J), (C,K)\big).$$

$$\begin{tikzcd}
 & (D_{B,\gamma}(J), \bar{J}, [\cdot]) \arrow[dr, dashed, "\exists!\ \text{PD-morphism}"] & \\
(B,J) \arrow[ur] \arrow[rr, "J \to K"] & & (C, K, \delta) \\
 & (A, I, \gamma) \arrow[ul] \arrow[uu, "\text{cptible}"'] \arrow[ur, "\text{cptible}"'] &
\end{tikzcd}$$

sketch of pf: $D_{B,\gamma}(J) \cong \Gamma_B(IB+J)/\sim$ all the relations you can think of.

w/ $\bar{J}$ = PD generated by image of J. (in degree 1).

Rmk. We always have $B/J \longrightarrow D_{B,\gamma}(J)/\bar{J}$. a section exists iff γ extends to B/J.] i.e. $\dfrac{D_{B,\gamma}(J)}{\bar{J}} \cong B/J$ iff γ extends to B/J

Note that γ automatically extends if I is principal or if $IB \subseteq J$.

II Let (A, I, γ)

$$\begin{array}{c} X_0 \\ \downarrow \text{l.f.p.} \\ S_0 = \mathrm{Spec}(A/I) \hookrightarrow \mathrm{Spec}(A). \end{array}$$

Defn/Prop. (Assume X_0/S_0 is separated, otherwise do hypercover.)

0) Choose affine covering $X_0 = \bigcup \mathrm{Spec}(A_i)$.

1) $\forall i$, choose smooth A-alg. B_i w/ $J_i \to B_i \twoheadrightarrow A_i$

2) $\forall i_0, i_1, \dots, i_k$, ~~form~~ we have $J_{i_0 \cdots i_k} \to B_{i_0} \otimes_A B_{i_1} \otimes_A \cdots \otimes_A B_{i_k}$

$$\downarrow$$

$$A_{i_0} \wedge A_{i_1} \wedge \cdots \wedge A_{i_k}.$$

and define $D_{i_0 \cdots i_k} = D_{B_{i_0 \cdots i_k}, \gamma}(J_{i_0 \cdots i_k})$.

3) Form the dR cplx $\Omega^\bullet_{D_\bullet/A}$.

$\underbrace{\text{Čech-dR double}}$

4) $H^i_{crys}(X_0/A, \mathcal{O}) := H^i(\mathrm{Tot}(\Omega^\bullet_{D_\bullet/A}))$.

which is well-defined!

Example. Let

$$\begin{array}{c} \mathbb{P}^1_k \\ \downarrow \\ \mathrm{Spec}\, k \hookrightarrow \mathrm{Spec}\, W_n(k) \end{array}$$

where k - perfect field of char. $p > 0$.

0) $\mathbb{P}^1_k = \bigcup \mathbb{A}^1_k$. $A_0 = k[x]$ $A_1 = k[y]$ $A_{01} = k[x^{\pm 1}]$.

1) $B_0 = W_n[x]$ $B_1 = W_n[y]$. $B_{01} = W_n[x, y]$.

2) $D_0 = W_n[x]$ $D_1 = W_n[y]$.

$$D_{01} = W_n[x, y, (xy-1)^{[r]}]/\sim \left(= W_n[x^{\pm 1}]\langle t \rangle \right).$$

call $xy - 1 = t$.
then $t^{p^n} = 0$, hence x invertible
& $y = x^{-1} \cdot (1 + t)$

$W_n\, x^{p^n-1} dx$
$W_n\, y^{p^n-1} dy$?

$W_n \oplus W_n \longrightarrow W_n$

Computation in Brian's talk essentially shows... $\frac{dx}{x}$!

III. The discussion of PD stuff globalizes mutatis mutandis.

$(S, \mathcal{I}, \gamma)$ ($\mathcal{I}$, qcoh sheaf of ideals)

$$\begin{tikzcd} V(\mathcal{J}) = X \arrow[r, hook, "\text{closed}"] \arrow[dr] & Y \arrow[d] \\ & S \end{tikzcd}$$

Defn. $D_{X,\gamma}(Y) := \underline{\mathrm{Spec}}_Y \, D_{\mathcal{O}_Y,\gamma}(\mathcal{J})$.

Rmk.

- So if γ extends to X, then

$$\begin{tikzcd} & X \arrow[r, hook, "j"] \arrow[dr] & D_{X,\gamma}(Y) \arrow[d] \\ X' \arrow[ur] \arrow[dr, hook] & & Y \\ & Y' \arrow[ur] \arrow[uur, dashed, "\exists"] & \end{tikzcd} \qquad \Big/ (S, \mathcal{I}, \gamma).$$

if closed w/ PD structure cptible w/ γ.

- $$\begin{tikzcd} & D_{X_0,\gamma}(X) \arrow[d] & \\ X_0 \arrow[r, hook] \arrow[ur] \arrow[d] & X \arrow[d] \\ S_0 \arrow[r, hook] & S \\ U \arrow[r, hook] \arrow[uu, "\text{open}", phantom] & T \arrow[u] \arrow[uu, dashed, bend left] \arrow[uuu, dashed, bend left=40] \end{tikzcd}$$

not necessarily cartesian, assume $S_0 \hookrightarrow S$ PD thickening. ~~and assume~~ $\boxed{\gamma \text{ extends to } X}$ and Assume further that X/S smooth.

b/c X/S sm.

PD thickening of affines

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assume $p^n \cdot \mathcal{O}_S = 0$ $(*)$ e.g. $W_n(k)$ X_0/S. (S, I, γ) PD-scheme. γ extends to X_0

Defn: $(X_0/S)_{crys}$: obj :

$$\begin{tikzcd} U \arrow[r, hook] \arrow[d, phantom, "\cap\,\text{open}"] & T \arrow[d] \\ X_0 \arrow[r] & S \end{tikzcd}$$

where $U = V(J)$, $J \subseteq \mathcal{O}_T$, (T, J, δ) PD cptible w/ γ.

note: $* \Rightarrow J$ is locally nilpotent $(\forall j \in J,\ j^{p^n} = 0)$.

morphism : $\begin{tikzcd} T' \arrow[r] & T \\ U' \arrow[u, hook] \arrow[r, hook, "\text{open}"] & U \arrow[u, hook] \end{tikzcd}$ $\quad (T', J', \delta') \longrightarrow (T, J, \delta)$ PD-morphism.

covering : $\{T_i \longrightarrow T\}$ is an open immersion & jointly cover $|T|_{Zar}$.

Caution: It doesn't have ~~enough~~ final obj.

Prop/Defn. A sheaf $\mathcal{F} \in Sh((X_0/S)_{crys}) \Leftrightarrow$

a law associating each (U, T, δ) w/ $\mathcal{F}_T \in Sh(T_{Zar})$ s.t.

$\forall \ (U', T', \delta') \xrightarrow{u} (U, T, \delta), \ \exists \ \rho_u : u^{-1}\mathcal{F}_T \longrightarrow \mathcal{F}_{T'}$

satisfying composition law

and if $T' \underset{\text{open}}{\hookrightarrow} T$, then ρ_u is an isom.

e.g. (1) $(U, T, \delta) \longmapsto \mathcal{O}_T$, denoted $\mathcal{O}_{X_0/S}$.

(2) $(U, T, \delta) \longmapsto \mathcal{O}_U$, denoted $i_* \mathcal{O}_{X_0}$.

(3) $0 \longrightarrow J_{X/S} \longrightarrow \mathcal{O}_{X_0/S} \longrightarrow i_*\mathcal{O}_{X_0} \longrightarrow 0$ exact.

$J_{X/S}(U, T, \delta) = \ker(\mathcal{O}_T \twoheadrightarrow \mathcal{O}_U)$.

Rmk. The prop/Defn above implies that $Sh((X_0/S)_{crys})$ has enough points.

Prop. (0) representable objects are sheaves.

(1) absolute finite product / inverse limit over finite nonempty index set of representable sheaves are representable.

(2) $\begin{tikzcd} X_0' \arrow[r, "g"] \arrow[d] & X_0 \arrow[d] \\ S' \arrow[r, "PD"] & S \end{tikzcd}$ $\rightsquigarrow$ get $g_{crys,*}$ & g^{-1}_{crys}

$Sh(X_0'/S') \longrightarrow Sh(X_0/S)$. morphism of topoi.

- $g^{-1}_{crys}((U, T, \delta))(U', T', \delta') = \mathrm{Hom}_{g\text{-PD}}((U', T', \delta'), (U, T, \delta))$.
- $g_{crys,*}(\mathcal{F})(U, T, \delta) = \mathrm{Hom}(g_{crys}(U, T, \delta), \mathcal{F})$

$= \Gamma((X_0'/S')_{crys}|_{(U,T,\delta)}, \mathcal{F}|_{(U,T,\delta)})$.

- $g_{crys}^{-1}(\mathcal{F})(U', T', \delta') = \operatorname*{colim}_{\{T' \to g_{crys}^{-1}(T)\}} \mathcal{F}(T)$.

(3) $$\begin{tikzcd} Sh((X_0)_{Zar}) \arrow[r, "i"] \arrow[rr, bend right, "id"'] & Sh((X_0/S)_{crys}) \arrow[r, "u"] & Sh((X_0)_{Zar}) \end{tikzcd}$$

i is induced by $(U, T, \delta) \longmapsto U$.

In particular $i_* \mathcal{O}_X (U, T, \delta) = \mathrm{Hom}_{crys}((U, T, \delta), i_* \mathcal{O}_X) = \mathrm{Hom}_{Zar}(U, \mathcal{O}_X)$
$= \mathcal{O}_X(U)$. Hence agrees w/ previous notation.

- $u_*(\mathcal{F})(U) = \Gamma((U/S)_{crys}, \mathcal{F})$.
- $u^{-1}(\mathcal{F})((U, T, \delta)) = E(U)$.

Thm. $$\begin{array}{ccc} X_0 & \xrightarrow{i} & X \\ \downarrow & \# & \downarrow \\ S_0 & \underset{V(I)}{\xrightarrow{PD}} & S \end{array}$$ Then $H^i((X/S)_{crys}, \mathcal{O}_{X/S}) \cong H^i((X_0/S)_{crys}, \mathcal{O}_{X_0/S})$.

We claim

pf: ① $i_{crys}^{-1} \underset{(U,T,\delta)}{\overset{\shortparallel}{T}}$ is represented by $(U \cap X_0 = U_0, T, \bar{\delta})$ in $(X_0/S)_{crys}$.

② $i_{crys,*}$ is exact

③ $i_{crys,*}(\mathcal{O}_{X_0/S}) \cong \mathcal{O}_{X/S}$.

④ Leray Spectral Sequence for $(\Gamma \circ i_{crys})$ implies what we want.

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④ we just have to verify $R\Gamma_{X/S}(i_{crys,*}\mathcal{F}) = R\Gamma_{X_0/S}(\mathcal{F})$

$$R\Gamma_{X/S}(i_{crys,*}\mathcal{F}) = R\mathrm{Hom}(e_{X/S}, i_{crys,*}\mathcal{F}) = R\mathrm{Hom}(i_{crys}^{-1} e_{X/S}, \mathcal{F}) = R\mathrm{Hom}(e_{X_0/S}, \mathcal{F}) = R\Gamma_{X_0/S}(\mathcal{F})$$

since $i_{crys}^{-1}(e_{X/S}) = e_{X_0/S}$ as it preserves finite limit & final obj. is empty limit.

①⇒③: $i_{crys,*}(\mathcal{O}_{X_0/S})(T)$
$= \mathrm{Hom}(i^{-1} h_T, \mathcal{O}_{X_0/S})$
$= \mathcal{O}_{X_0/S}((U_0, T, \bar{\delta})) = \mathcal{O}_T(T) = \mathcal{O}_{X/S}(T)$.

①⇒②: follows similarly. (i^{-1} doesn't change the "T"-part, and Crys. sheaf has a Zariski interpretation).

①: by definition

Now we're ready to prove the famous comparison thm to de Rham coh. assuming appearance of a smooth lift.

Let us fix notations.

$$\begin{tikzcd} X_0 \arrow[r, hook] \arrow[d] \arrow[dr, phantom, "\#"] & X \arrow[d, "\text{smooth}"] \\ S_0 \arrow[r, hook, "PD", "V(I)"'] & S \end{tikzcd}$$

- $$\begin{tikzcd} & \underline{\mathrm{Spec}}(D_X(1)) \arrow[d] & \\ X \arrow[ur] \arrow[r, hook] \arrow[dr] & X \times_S X \arrow[d] & \\ & S & \end{tikzcd}$$ $\underline{\mathrm{Spec}}(D_X(1)) \leftarrow$ PD-hull/envelope.

  (étale) locally $D_X(1)$ is divided power alg. over X w/ rel. dim(X/S) many ~~var~~ variables.

Fact (To be explained in the next 2 talks): $\mathcal{O}_{X_\bullet/S} \xrightarrow{qis} L(\Omega^\bullet_{X/S}) := D_X(1) \otimes_{\mathcal{O}_X} \Omega^\bullet_{X/S}$.

Cautions: The connecting morphism on the RHS is subtle.

e.g. $S = (A, I, \gamma)$. $X = \mathrm{Spec}\, A[x]$. Then $D_X(1) = A[x]\langle t\rangle$.

We have exact sequence $0 \to A[x] \to A[x]\langle t\rangle \to A[x]\langle t\rangle\, dx \to 0$

(this only verifies on $(X, X, 0)$...).

$x \longmapsto x - t$

$x^n t^{[m]} \longmapsto (n x^{n-1} t^{[m]} + x^n t^{[m-1]})dx$

- Similarly we have $$\begin{tikzcd} & \underline{\mathrm{Spec}}(D_X(\nu)) \arrow[d] & \\ X \arrow[ur] \arrow[r, hook] \arrow[dr] & \underbrace{X \times_S X \times_S \cdots \times_S X}_{\nu+1\text{-copies of } X} \arrow[d] & \\ & S & \end{tikzcd}$$

$\boxed{\text{! not relevant...}}$

The sheaf $h_X =: \tilde{X}$ has the property (due to smoothness of X/S) that $\tilde{X} \longrightarrow e$ is a covering/epimorphism.

Also $\tilde{X} \times_e \tilde{X} \times_e \cdots \times_e \tilde{X} = \underline{\mathrm{Spec}}(D_X(\nu))$.

Hence for any (bounded) cplx $K^\bullet$ in $Sh((X/S)_{\mathrm{crys}})$, we ~~may use the double cplx~~ have

$$K^\bullet \xrightarrow{qis} \left(Rj_{\tilde{X},*}\, j_{\tilde{X}}^{-1} K^\bullet \longrightarrow Rj_{\tilde{X}^2,*}\, j_{\tilde{X}^2}^{-1} K^\bullet \longrightarrow \cdots\right) \quad \text{(notation to be explained)}$$
$$=: \check{C}A^\bullet_{\tilde{X}}(K^\bullet).$$

We need the following

key Lemma (localization of crystalline topos) $\mathcal{Z} = (V, Z, \varepsilon) \in (X_0/S)_{crys}$. We have a morphism of topoi

$$Sh(X_0/S)_{crys}|_{\tilde{\mathcal{Z}}} \xrightarrow{j_{\mathcal{Z}}} Sh(X_0/S).$$

Assume $V = X_0$, then ① $j_{\mathcal{Z},*}$ is exact

② $j_{\mathcal{Z},*}E$ is acyclic for ~~$u_{X/S*}$~~ u_* where.

$$\begin{tikzcd} Sh(X_0/S)|_{\tilde{\mathcal{Z}}} \arrow[r, "\varphi"] \arrow[d, "j_{\mathcal{Z}}"'] \arrow[dr, phantom, "\curvearrowright"] & (Z)_{Zar} \arrow[d, "\cong"] \\ Sh(X_0/S) \arrow[r, "u"] & (X_0)_{Zar} \end{tikzcd}$$

pf: $(j_{\mathcal{Z},*}E)(U,T,\delta) = p_{T,*}(E_{p_Z})$ where

$$D_{U\cap V,\delta,\varepsilon}(T \times_S Z) =: \text{"}\tilde{T} \times_e \tilde{Z}\text{"} \quad \xrightarrow{p_T} T, \quad \xrightarrow{p_Z} Z$$

$\quad \| \quad \uparrow$
$\quad U$

and 1) p_T is a homeomorphism.

2) $E \to F \to G$ exact iff $E_T \to F_T \to G_T$ exact $\forall T$.

1) + 2) $\Longrightarrow$ ①.

as for ②: φ_* is exact. Hence $(u \circ j_Z)_*$ is exact.

But now $j_{Z,*}$ is already exact,

Therefore by Leray S.S. argnement, $Ru_* j_{Z,*} = u_* j_{Z,*}$.

Back to the comparison, (we apply to $(X/S)_{crys}$...) where $Z =$ ~~Spec D(?)~~ X.

$$Ru_*(\mathcal{O}_{X_0/S}) \xrightarrow[\sim]{\text{Fact}} Ru_*(L\Omega^\bullet_{X/S}) \to \cancel{(D_X(U) \otimes \Omega^\bullet_{X/S})}$$

~~key Lemma Fact~~ key Lemma

qis $\quad \Omega^\bullet_{X/S}$ ~~qis~~

where we have used that $L(E) = j_{X,*}\varphi^* E$
c.f. [BOgus, Prop. 6.10].

~~where in the last step, we have used:~~

~~fixing k: $\Omega^k_{X/S} \xrightarrow[\text{qis}]{\sim} \left(D_X(1) \otimes_X \Omega^k_{X/S} \longrightarrow D_X(2) \otimes_X \Omega^k_{X/S} \longrightarrow \cdots \right)$.~~

~~(c.f. [Bhatt–dJ] Lemma 2.17)~~

Apply $R\Gamma(X_{Zar}, -)$ to both sides, we get the following thm.

Thm
$$\begin{array}{ccc} X_0 & \longrightarrow & X \\ \downarrow & \# & \downarrow \text{sm.} \\ S_0 & \hookrightarrow & S \end{array}$$
we have $H^i((X_0/S)_{crys}, \mathcal{O}_{X_0/S}) \cong H^i_{dR}(X/S)$.

Back to Faltings's Defn:

$\left\{ \coprod_i (\operatorname{Spec} A_i, \operatorname{Spec} B_i, \tilde{\gamma}) = T_i \right\} \longrightarrow e$ forms a covering/epimorphism in $Sh((X_0/S)_{crys})$

w/ $\prod_{i \in \{i_0, \dots, i_k\}} \tilde{T}_i = \left(\bigcap_I \operatorname{Spec} A_i, \operatorname{Spec} D_{B_{i_0 \cdots i_k}, \gamma}(J_{i_0 \cdots i_k}), \bar{\gamma} \right)^{\sim}$.

$\qquad \overset{\shortparallel}{I}$

Hence $\left(R\Gamma((T_i/S)_{crys}, \mathcal{O}_{X_0/S}) \longrightarrow R\Gamma((T_{ij}/S)_{crys}, \mathcal{O}_{X_0/S}) \longrightarrow \cdots \right)$

computes $R\Gamma((X_0/S)_{crys}, \mathcal{O}_{X_0/S})$.

Then we use [Bhatt–dJ] Thm 2.12.

$$R\Gamma((\tilde{T}_I/S)_{crys}, \mathcal{O}) \simeq (D \longrightarrow \Omega^1_D \longrightarrow \Omega^2_D \longrightarrow \cdots).$$

where notations have to be understood appropriately...